Graph Sketching Using Derivatives

1. Sketch a graph of a differentiable function \( f(x) \) over the closed interval \([-2, 7]\), where \( f(-2) = f(7) = -3 \) and \( f(4) = 3 \). The roots of \( f(x) = 0 \) occur at \( x = 0 \) and \( x = 6 \), and \( f(x) \) has properties indicated in the table below:

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -2 < x < 0 & 0 < x < 2 & x = 2 & 2 < x < 4 & x = 4 & 4 < x < 7 \\
\hline
f'(x) & \text{positive} & 0 & \text{positive} & 1 & \text{positive} & 0 & \text{negative} \\
f''(x) & \text{negative} & 0 & \text{positive} & 0 & \text{negative} & 0 & \text{negative} \\
\hline
\end{array}
\]

2. Sketch a graph of the continuous even function \( g(x) \) over the closed interval of \( x \) values \([-5, 5]\) having a range of \( g(x) \) values \([-1, 0]\). For \( x \geq 0 \), roots of \( g(x) = 0 \) occur at every whole number \( k \) and roots of \( g'(x) = 0 \) occur at \( \frac{k}{2} \). The first and second derivatives of \( g(x) \) exist everywhere except at \( x = k \). Furthermore, \( g''(x) > 0 \) for every \( x \neq k \).
3. Sketch a function \( h(x) \) from the following information:

(a) \( h(-x) = -h(x) \)

(b) \( \lim_{{x \to 0^+}} h(x) = \infty \)

(c) \( \lim_{{x \to +\infty}} h(x) = 0 \)

(d) For \( x > 0 \), \( h(x) = 0 \) only at \( x = 1 \)

(e) For \( x > 0 \), \( h'(x) = 0 \) only at \( x = 2 \)

(f) For \( x > 0 \), \( h''(x) = 0 \) only at \( x = 3 \)

Concept Connectors

4. The graph of \( f(x) \) is shown on the closed interval \([-6a, 6a]\):

Answer the following questions regarding \( f(x) \):

(a) For \( x \neq 0 \), the graph of \( f(x) \) has symmetry about the ________________.

(b) \( f \) has point(s) of discontinuity at \( x = ________________ \).

(c) \( \lim_{{x \to 0}} f(x) = ________________ \).

(d) The zeros of \( f(x) \) occur at \( x = ________________ \).

(e) \( f'(x) \) does not exist at \( x = ________________ \).

(f) \( f''(x) < 0 \) for the \( x \) interval(s) ________________.